Problem of identification of nonlinear dynamic models for diagnostic task

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Abstract. Physical phenomena accompanying destruction processes of technical systems are nonlinear and "low-energetic" by nature, while during wearing out mainly a nonlinear disturbance changes. Out of many inference techniques – on the observation basis of the state of the system – the best one is undoubtedly the well-defined mathematical model, allowing inferring "backwards" and "forward", which means finding the genesis and prognosis of the phenomenon. However, such model should be nonlinear. Problems related to the identification of nonlinear dynamic models in a frequency domain and a proposition of solving this problem for the needs of technical diagnostics, i. e. in situations when the observed wear out effects are significantly smaller than the dynamic effects – are discussed in the hereby paper. The bases of the proposed method constitute the discussion of possible solutions of a certain class of nonlinear differential equations and resulting from that statements on the possibility of nonlinear disturbance approximations by a series of the selected harmonic frequencies.

Keywords: nonlinear model, model identification, technical diagnostics.

1. Introduction

The needs of the vibroacoustic diagnostics require to have the model describing the signal propagation path from the place of the expected defects to the observation point. Thus, the selected part of the machine system and not the entire system can be described in details. Inputs and outputs from the object under modelling should be indicated from the observation (measurement) [1]. Let us adopt the following assumptions:

– The object able for operations is described, with a good accuracy, by a set of ordinary linear differential equations of the second order;

– Degradation of the system introduce a nonlinear disturbance, which is changing during further wear (failure).

2. Model identification

The linear system can be easily reduced to the main coordinates in which equations are uncoupled:

$$\ddot{\xi}_i + \omega_{0i}^2 = p_i(t), \tag{1}$$

where ω_{0i} – the ith natural frequency.

With coordinates defined in this nonlinear system ξ obviously stays nonlinear but its left sides get de-conjugate and it finally appears as:

$$\ddot{\xi}_i + \omega_{0i}^2 = \mathbf{F}(\xi_i; \dot{\xi}_i; t), \tag{2}$$

where: ξ_i, $\ddot{\xi}_i$ – vector of normal coordinates and their second derivatives, ω_{0i} –vector of natural frequency of linearized system, $\mathbf{F}$ – matrix of nonlinear functions of normal coordinates and of their first derivatives and time.

Such system can be solved only in an approximate way. There is only one possibility of predicting the form of the solution on the basis of the generalised approximate analytic solution for small number of the degrees of freedom. In the first approximation the following form is obtained (the autor applied here the KB method, however the identical form can be obtained when applying e.g. variance methods (Ritz'es, Galerkin's)):

$$\xi_j = a_j(t)\cos\psi_j(t) + \frac{1}{(2\pi)^{1+n}}\sum_n\sum_m \frac{\exp i(m\Omega + \sum_k \psi_k n_k)}{\omega_{ok}^2 - (m\Omega^2 + \sum_k n_k\omega_{ok}^2)}$$
$$\cdot \underbrace{\int_0^{2\pi}\int_0^{2\pi}\int_0^{2\pi}\dots\int_0^{2\pi}}_{n+1} f_k \exp\left\{(-i)\left[m\Omega t + \sum_k n_k\psi_k\right]\right\} d\psi_1 \dots d\psi_n dt, \tag{3}$$
$$\bigwedge_{m,n,k \in \left\{N - (m;n;k):\, \omega_{ok}^2 - (m\Omega^2 + \sum_k n_k\omega_{ok}^2) = 0\right\}},$$

where f_k is as follows:

$$f_k = \Phi_k(a_1\cos\psi_1, \dots, a_k\cos\psi_k;\ -a_1\omega_{o1}\sin\psi_1, \dots, -a_k\omega_{ok}\sin\psi_k;\ \Omega t). \tag{4}$$

For the sake of readability it was assumed, that all excitations are polyharmonic of the commensurable frequencies and the common measure of these frequencies were denoted by Ω (an assumption of an arbitrary number of incommensurable frequencies does not change the solution form but only multiplication factor of integrations).

When vibrations of a certain harmonic group do not occur in the observed spectrum, one is permitted to state the zeroing criteria of coefficients of these components. These criteria identify coefficients of power series expansion of nonlinear characteristics of elasticity and damping in a relatively simple way, according to:

$$A_{m_o,n_o,k_o} = 0 \Leftrightarrow \underbrace{\int_0^{2\pi}\int_0^{2\pi}\dots\int_0^{2\pi} f\exp(-i)\left[m_o\Omega + \sum n_{k_o}\psi_{k_o}\right] d\psi_1 \dots d\psi_n dt = 0}_{n+1} \tag{5}$$
$$\Rightarrow \quad K(k_1, \dots, k_n;\ m_1, \dots, m_n;\ c_1, \dots, c_n) = 0.$$

However, regardless of the possibility of finding dependencies between system parameters on the basis of the occurrence or not occurrence of components of system responses of various frequencies, this is not the explicit inverse identification method of elasticity and damping. The solution Eq. (3) is only pertinent far from the resonance zones. The condition Eq. (5) is only the necessary condition of the spectrum range of the solution and does not give any information concerning the amplitudes (!).

However, the form of the solution Eq. (3) is an additive one. An influence of a nonlinear disturbance is written in the form of a series of successive corrections (each successive correction has the same form and corrects only functions f_k). Thus, the dependence: signal ↔ model for the systems with the nonlinear disturbance can be written:

$$S_t\{y(t,n,r,\theta)\} = \sum p_i(t) * h_i(t-\tau) + \varphi + \psi, \tag{6}$$

where: S_t – selection operator in the time domain, φ – vector of correction functions, which brings the linear system solution into the nonlinear one, ψ – measurement noise.

Since functions φ are here functions of a single variable the equation can be brought into a frequency domain by applying the Fourier transform:

$$S_\omega \Im S_t\{y(t,n,r,\delta)\} = \sum P_i \cdot H_i + \Phi + \Psi, \tag{7}$$

where Fourier transforms of values from Eq. (6) are marked by capital letters, and S_ω denotes the selection operator in the frequency domain (filtration operator).

The sought after form of solution enables to identify the diagnostic model, according to the following procedure: z_i denotes parameters of the linear model at the „zero" ($\theta = \theta_0$) state, which are difficult to the direct determination. The simple condition of the parametric identification of the linear system are built on the basis of Eq. (7):

$$\begin{gathered}\left|S_\omega \Im S_t\{y(t,n,r,\theta_0)\} - \sum P_i \cdot H_i\,(z_1,\dots,z_n,\dots)\right| < \delta, \\ \Downarrow \\ z_i\end{gathered} \tag{8}$$

where δ denotes the permissible identification error, when $\delta > \psi$.

For the other selected state θ_k:

$$\begin{gathered}\left|S_\omega \Im S_t\{y(t,n,r,\theta_k)\} - \sum P_i \cdot H_i\,(z_1,\dots,z_n,\dots) + \Phi_k\right| < \delta, \\ \Downarrow \\ \varphi_k(\theta) = \Im^{-1}\Phi_k.\end{gathered} \tag{9}$$

In order to make the model diagnostically useful one other empirical verification has to be done for $\theta_n \neq \theta_k$. In case of large discrepancies functions θ_k should be corrected. Such procedure leads, generally, to positive results.

3. Diagnostic task

The presented hereby proposition of considering the evolution of the mechanical systems as the increase of the nonlinear disturbance together with its two stage identification process, (where in the first stage the qualitative analysis is performed and in the second stage the quantitative determination of correction functions is done), reduced itself to several detailed applications [1, 3, 4].

Finally the diagnostic task, at such way of modelling, reduces to the following formalism:

$$\begin{aligned}&Y(\omega,\Theta_m) = \sum P_k(\omega)\cdot H_k(q_i,z_i) + \Phi^*(\Theta_m) + \psi, \\ &Y(\omega,\Theta_n) = \sum P_k(\omega)\cdot H_k(q_i,z_i) + \Phi^*(q_i,\Theta_n) + \psi, \\ &\Downarrow \\ &\Delta_{mn}Y(\omega) = -\Phi^*(\Theta_m) + \Phi^*(q_i,\Theta_n) + \psi = \Delta\Phi^*(q_i,\Theta_m,\Theta_n,\psi), \\ &\Downarrow \\ &q_i = \Delta\Phi^{*)-1}\Delta_{mn}X(\Theta_m,\Theta_n,\psi),\end{aligned} \tag{10}$$

where q_i is the sought after "initial" defect parameter.

4. Comments and conclusions

It is obvious that the key problem is the existence of an inverse transform ($\Delta\Phi^{*)-1}$), which – in practice – means selecting a measure allowing to differentiate vibrations spectra of systems with disturbances from the systems without them. Since from the preliminary assumptions it results, that Eq. (10) are differentiable at least two times, the task can be solved for displacements, speeds or accelerations of vibrations in the frequency domain. This would mean that this final reasoning is only confirming the obvious fact, that the spectra of the systems – with and without defects – are different.

However, it is not so. In the first place, for the considered case, the expected changes caused by a defect concern nonlinear phenomena and the observable changes of the general level of vibrations or amplitudes of main harmonic components should not be expected. In the second place, an empirical finding of function φ_i allows to create the mathematical model given by nonlinear differential equations, in which one of the parameters will be the defect parameter selected by us, it means change of the local elasticity coefficient, which – in turn – means solving the inverse task of the dynamic model identification.

To be fully exact, it should be mentioned, that this model will not be identical to the model assumed at the beginning, but will be a nonlinear model of solutions identical to the results of investigations obtained with the same accuracy. Since this problem is far above the scope of this elaboration we will not dwell any deeper into it.

Now, let us think what should be measured and which measure (distance) sensitive to weak (!) changes of spectrum characteristics should be selected.

There are several methods of detecting nonlinear phenomena in the spectrum of vibration accelerations, e.g. multi – spectral analysis. However, in our case we will try to determine very weak effects related rather to high harmonic vibrations and in addition in a very narrow frequency range. It would be also advantageous if we could build amplitude measures in the real numbers domain without considering the phase relations.

This last task is rather complicated. In the author's opinion the coherence methods [5] or the proposed by professor Batko reduction of the task into the stability analysis of phase trajectories [6] – are functioning well.

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